LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – **NOVEMBER 2022**

17/18UMT5MC01 – REAL ANALYSIS

Date: 23-11-2022 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

PART - A

Answer ALL Questions **(10 x 2 = 20)**

1. Define countable, give an example.
2. State Archimedean property.
3. Define closed set.
4. Define accumulation point.
5. Give an example to show every continuous function need not be uniformly continuous.
6. State uniform continuity theorem.
7. Define differentiability at a point.
8. Define local maximum, give an example.
9. Define total variation.
10. Prove that every function defined and monotonic on a bounded closed interval is of bounded variation on that interval.

PART - B

Answer any FIVE Questions **(5 x 8 = 40)**

11. Prove that every subset of a countable set is countable.
12. Prove that e is irrational.
13. Let $M = R^n$ and let $x = (x_1, x_2 \dots x_n), y = (y_1, y_2 \dots y_n)$ and $z = (z_1, z_2 \dots z_n) \in R^n$.Define $d(x, y) = \left\{\sum_{k=1}^{n}\left(x_k\text{-}y_k\right)^2\right\}^{\frac{1}{2}}$. Prove that (M, d) is a metric space.
14. State and prove Lagrange's mean value theorem.
15. If f and g are both continuous at a point $x_0 \in X$, then prove that $f + g, fg$ and kf are continuous a x_0, where k is a constant.
16. Prove that every convergent sequence is a Cauchy sequence.
17. Let (X, d) be a metric space. Then prove that the following (i) the union of an arbitrary collection of open sets in X is open in X (ii) the intersection of an arbitrary collection of closed sets in X.
18. Show that every compact subset of a metric space is complete.

PART - C

Answer any TWO Questions **(2 x 20 = 40)**

19. Prove that every bounded and infinite subset of R has atleast one accumulation point.

20. (a) State and prove Cauchy Schwartz inequality.

    (b) Explain about compact set and complete metric space. **(12+8)**

21. (a) State and prove Taylor's Theorem.

    (b) State and prove Rolle's theorem. **(12+8)**

22. Let f be of bounded variation on $[a, b]$ and $c \in (a, b)$. Prove that f is of bounded variation on $[a, c]$ as well as on $[c, b]$ and $V_f[a, b] = V_f[a, c] + V_f[c, b]$.

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